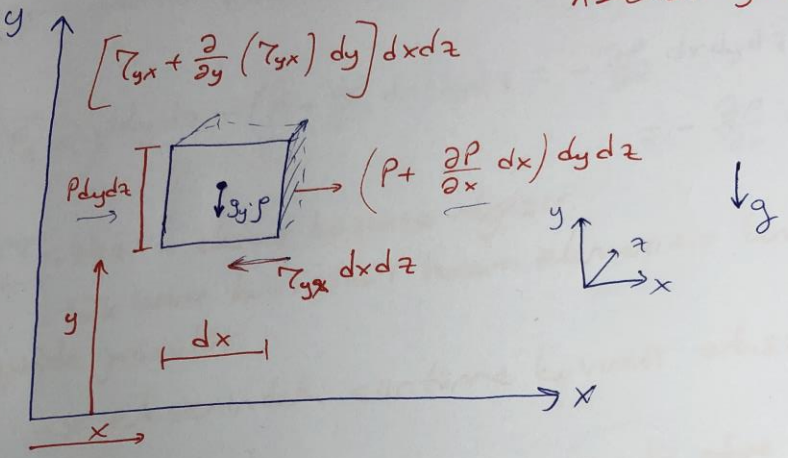
MOMENTUM DENKLEMLERİ

. Sınır tabaka içinde akışkan partikülüre sürtünme kuvvetleri etki eder.

$\boxed{\text{• Denklem takımı}}$ ← Akışkan hareketini etkileyen dinamik özelikler.

. Hareket denklemleri
. Momentum denklemleri

Newton'un 2. hareket yasası.



Atalet kuvvetleri → dış kuvvetler (tüm partiküller)
Dinamik kuvvetler → yüzey kuvvetleri (Basınç)
Sürtünme kuvvetleri

Atalet kuvvetleri
- yerçekimi
- Savurma kuvvetleri
- Emanyetik alan

- Sürtünme kuvvetleri (Akışkanın iç sürtünmesi)

-Yerçekimi kuvveti hacim elemanının merkezi

$$d\vec{F_1} = g_y \, dxdydz \, \rho = g_y \, \rho \, dV \qquad \left(1kg \frac{m}{s^2} = 1N\right)$$

$\frac{m}{s^2} \frac{kg}{m^3} m^3$

$P \, dy \, dz$

$-\left(P + \frac{\partial P}{\partial x} dx\right) dy \, dz$

Hacim elemanı üzerine etki eden net basınç kuvveti

$d\vec{F_2}$

$$d\vec{F_2} = P dy dz - \left(P + \frac{\partial P}{\partial x} dx\right) dy dz = -\frac{\partial P}{\partial x} dx dy dz$$

$$= -\frac{\partial P}{\partial x} dV$$

NOT: -Hız x ekseni boyunca değişir.

- Sürtünme kuvvetleri hacim elemanının kenar yüzeyinde görülür.

-x eksenindeki sürtünme kuvveti akışa terstir.

$\tau_{yx} dx dz$ $\quad$ x+dx ~~akışa~~ kesitinde

$\left[\tau_{yx} + \frac{\partial}{\partial y}(\tau_{yx}) dy\right] \underset{\text{Alan}}{\underline{dx dz}}$ sürtünme kuvveti akış istikametinde

Net Sürtünme Kuvveti

$$d\vec{F_3} = \left[\tau_{yx} + \frac{\partial}{\partial y}(\tau_{yx}) dy\right] dx dz - \tau_{yx} dx dz$$

$$= \frac{\partial}{\partial y}(\tau_{yx}) dy dx dz = \frac{\partial}{\partial y}(\tau_{yx}) dV$$

$$d\vec{F_3} = \frac{\partial}{\partial y}(\tau_{yx}) dV$$

$$\boxed{\tau_{yx} = \mu \frac{d\vec{V}}{dy}}$$

Toplam kuvvet:

$$d\vec{F} = d\vec{F_1} + d\vec{F_2} + d\vec{F_3}$$

$$d\vec{F} = \left[\rho g_y - \frac{\partial P}{\partial x} + \mu \frac{\partial P}{\partial y}\left(\frac{d\vec{V}}{dy}\right)\right] dv$$

$$\boxed{d\vec{F} = \left(\rho g_y - \frac{\partial P}{\partial x} + \mu \frac{\partial^2 \vec{V}}{\partial y^2}\right) dv}$$

$$d\vec{F} = \frac{d\vec{M}}{dt} = \frac{d}{dt}(m\vec{V}) = m\vec{a}$$

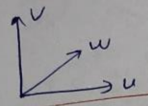
$$\boxed{\vec{a} = \frac{d\vec{V}}{dt}}$$

$$d\vec{F} = m\frac{d\vec{V}}{dt} = \rho\, dv \frac{D\vec{V}}{dt}$$

$$\rho \frac{D\vec{V}}{dt} = \rho g_y - \frac{\partial P}{\partial x} + \mu \frac{\partial^2 \vec{V}}{\partial y^2}$$

$\vec{V}$ — u, v, w

$$\rho \frac{D\vec{V}}{dt} = \rho g_y - \frac{\partial P}{\partial x} + \mu \frac{\partial^2 \vec{V}}{\partial y^2}$$



$$\rho \frac{Du}{dt} = \rho g_x - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \frac{Dv}{dt} = \rho g_y - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \frac{Dw}{dt} = \rho g_z - \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

Navier Stokes denklemleri

zamana göre hız bileşenlerinin toplam türevi

x, y ve z

$\rho\, dx\, dy\, dz$

x yönündeki ivmesi

$$\frac{Du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$\frac{Dv}{dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$\frac{Dw}{dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

$$\rho\left(\frac{Du}{dt}+\frac{Dv}{dt}+\frac{Dw}{dt}\right)=\rho(g_x+g_y+g_z)-\left(\frac{\partial p}{\partial x}+\frac{\partial p}{\partial y}+\frac{\partial p}{\partial z}\right)$$

$$+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

$$\boxed{\rho\left(\frac{D\vec{V}}{dt}\right)=\rho\vec{g}-\nabla p+\mu\left(\nabla^2\vec{V}\right)}$$

$\nu=\frac{\mu}{\rho}$

$$\frac{D\vec{V}}{dt}=\vec{g}-\frac{1}{\rho}\nabla p+\nu\left(\nabla^2\vec{V}\right)$$

$\vec{g}\to 0$

Eğer, zorlanmış taşınım ısı transferi

$Ri=\frac{Gr}{Re^2} < 1$

$$\boxed{\frac{D\vec{V}}{dt}=-\frac{1}{\rho}\nabla p+\nu\left(\nabla^2\vec{V}\right)}$$

y yönündeki viskoz kayma gerilmeleri ihmal edilirse,

$$\rho\left(\frac{Du}{dt}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}$$

$$\rho\left(\frac{Du}{dt}\right)=\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial x}$$

↑ Basitleştirilmiş momentum denklemi